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SOME STANDARD DISTANCES FOR CIRCULAR PYTHAGOREAN FUZZY SETS

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Abstract
This article presents four standard distances for Circular Pythagorean Fuzzy Sets (C-PFSs). These sets are extensions of the standard PFS that are inturn extensions of Zadeh's fuzzy sets. It is obivious that the distance measures in C-PFS environment are different than those for the standard PFSs. Suitable Illustrations for the proposed distances and their comparison with distance measures of PFSs are provided.

Keywords: Pythagorean Fuzzy Sets, Circular Pythagorean Fuzzy Sets, Distances for Circular Pythagorean Fuzzy Sets

1. Introduction:

The Intuitionistic Fuzzy Set (IFS) concept introduced in 1983 by Krassimir Atanassov was the extension of Zadeh's fuzzy set [17]. On the other hand, the Pythagorean Fuzzy Sets (PFS) is also an object of different extension of fuzzy set theory. PFS is a new tool developed to deal problems with vagueness by considering the sum of the membership grade, μ and non-membership grade, ν with $\mu+\nu \geq 1$ and sum of their squares not exceeding one. One such extension paves way for Circular PFS (C-PFS, see [4]). Here, we extended the region of the radius r values of the objects of PFS to be $[0,\sqrt{2}]$ because we would like the points in the center $\langle 0,1\rangle$ and $\langle 1,0\rangle$ to be able to cover the whole PFS triangle, which can be valid for if $r \geq 2$. When $r = 0$, the C-PFS represents the standard PFS. Also, when $r > 0$, the C-PFS is an object different from the ordinary PFS. In reality, in ordinary PFS theory, this is a way to represent the existence of circles around each elements of universe E (Figures 2). The concept of C-PFS and their distances provides a kind of visualization that can model areal life problem with better clarity and also provides a best solution for certain problems in nature.

The rest of the manuscript is structured as follows. Fundamentals related to C-PFSs are covered in "Preliminaries". Four standard distances between Circular Pythagorean Fuzzy sets are defined in section 2. Section 3 describes the illustrations for the defined distances and a Comparative analysis is put forward to examine the importance of the newly defined distances. Finally, the concluding notes of the article are presented in section 4.

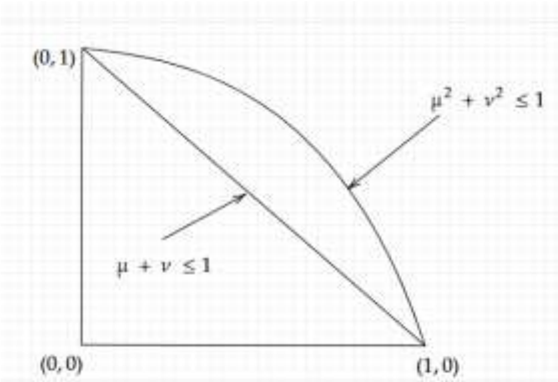

Figure 1. Geometrical interpretation of an element of an IFS and PFS.

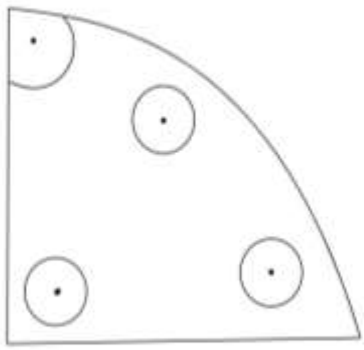

Figure 2. Geometric representation of the elements of a C- PFS

2. Preliminaries:

This section recalls about some basic concepts and definitions related to Circular Pythagorean Fuzzy Sets that are required for the study of this paper.

Definition 1[1]: A **Fuzzy set** A in X (Set of real numbers) is a set of ordered pairs

$$A= \{\langle x, \mu_A(x)/x \in X\rangle\}$$

is called membership function of x in A which maps X to [0,1].

Definition 2[1]: Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A in X is an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$$

Here, $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$ are defined as the degrees of membership and non-membership of the element x to the IFS A. For each IFS, the intuitionistic index or hesitancy degree of x in X to the IFS A is defined as

$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Definition 3[8]: Let M be a fixed set. Then, **a Pythagorean fuzzy set (PFS)** in M is defined as given below:

$$P = \{(m, \mu_P(m), \nu_P(m)) | m \in M\},$$

where $\mu_P(m)$ and $\nu_P(m)$ are mappings from M to [0, 1], with conditions $0 \leq \mu_P(m) \leq 1$, and $0 \leq \nu_P(m) \leq 1$ and also $0 \leq \mu^2_P(m) + \nu^2_P(m) \leq 1$, for all $m \in M$, and they denote the degree of membership and non-membership of element $m \in M$ to set P, respectively. Here, $\mu_P + \nu_P \leq 1$ or ≥ 1 , $\pi_P(m) = 1 - \mu^2_P(m) - \nu^2_P(m)$, which is called the Pythagorean fuzzy index of element $m \in M$ to set P, representing the degrees of indeterminacy of M to P. Also, $0 \leq \pi_P(m) \leq 1$, and $\pi^2_P(m) + \mu^2_P(m) + \nu^2_P(m) = 1$, for every $m \in M$.

Definition 4: A **metric defined** on a set X is a function $d : X \times X \to R$ with the following three properties:

1. $d(x, y) \geq 0$ for all $x, y \in X$, and equality holds iff $x = y$ (positivity).
2. $d(x, y) = d(y, x)$ for all $x, y \in X$ (symmetry).
3. $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$ (the triangle inequality).

Here, $d(x, y)$ represents the **distance** between x and y for the **metric space** (X, d). It is evident that d must satisfy the properties.

Definition 5: Let $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in R^n$. Then, some standard distances over sets are as defined follows:

1. Euclidean distance: $\boldsymbol{d_2(x,y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}}$
2. Manhattan (Hamming) distance:

$$d_1(x,y) = \sum_{i=1}^{n} | x_i - y_i |$$

Definition 6: The four distances over PFSs are defined as follows:

Let any two Pythagorean Fuzzy Sets A and B are defined as follows:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$
$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\},$$

where μA, νA, μB, νB : $E \rightarrow [0,1]$ and ${\mu_A}^2(x) + {\nu_A}^2(x) \leq 1, {\mu_B}^2(x) + {\nu_B}^2(x) \leq 1$ for each $x \in E$. Let $\mathcal{C}_E$ be the cardinality of universe E throughout this paper.
The following distances are described:

$$H_2(A,B) = \frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}(|\mu_A - \mu_B| + |\nu_A - \nu_B|), \quad (1)$$

(Pythagorean fuzzy Hamming distance)

$$E_2(A,B) = \sqrt{\frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2)}, \quad (2)$$

(Pythagorean fuzzy Euclidean distance)

$$H_3(A,B) = \frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}(|\mu_A - \mu_B| + |\nu_A - \nu_B| + |\pi_A - \pi_B|), \quad (3)$$

(Szmidt and Kacprzyk's form of Pythagorean fuzzy Hamming distance)

$$E_3(A,B) = \sqrt{\frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2 + (\pi_A(x) - \pi_B(x))^2)},$$
(4)

(Szmidt and Kacprzyk's form of Pythagorean fuzzy Euclidean distance).

Definition 7[6]: Circular PFS is defined as follows:

Let E be a fixed universe and A be its subset. The set $A_r{}^* = \{\langle x, \mu_A(x), \nu_A(x); r\rangle | \ x \in E\}$, where $0 \leq {\mu_A}^2(x) + {\nu_A}^2(x) \leq 1$ of the circle around each element $x \in E$, is called a C-PFS and functions $\mu_A(x): E \rightarrow [0,1]$ and $\nu_A(x): E \rightarrow [0,1]$ represent the degree of membership (validity, etc.) and non-membership (non-validity, etc.) of element $x \in E$ to a fixed set $A \subseteq E$. Now,we can define also function $\pi_A : E \rightarrow [0, 1]$ by means of $\pi_A(x) = \sqrt{1 - {\mu_A}^2(x) - {\nu_A}^2(x)}$ and it corresponds to degree of indeterminacy (uncertainty, etc.). Let us remark that in [4], the radius r was defined to take values from the interval [0,1].

3. Distances over Circular Pythagorean Fuzzy Sets:

This section, introduces distances over any two C-PFSs. Basic ideas of norms, metrics and distances over PFSs were proposed in [8] and [9], where the first two distances were considered. The extensions of the first two distances were given as next two distances in [10] by E. Szmidt and J. Kacprzyk. Many other distances were introduced over PFSs in the later years.

Definitions of the First Four Distances over C-PFSs

Here, we introduce the following four distances for C-PFS that are modifications of distances (1)–(4) as follows:

Circular Pythagorean Fuzzy Hamming distance:

$$H_2(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}(|\mu_A - \mu_B| + |\nu_A - \nu_B|)\right) \quad (5)$$

Circular Pythagorean Fuzzy Euclidean distance:

$$E_2(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \sqrt{\frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2)}\right) \quad (6)$$

Szmidt and Kacprzyk's form of Circular Pythagorean Fuzzy Hamming distance:

$$H_3(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}(|\mu_A - \mu_B| + |\nu_A - \nu_B| + |\pi_A - \pi_B|)\right) \quad (7)$$

Szmidt and Kacprzyk's form of Circular Pythagorean Fuzzy Euclidean distance:

$$E_3(A,B) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + \sqrt{\frac{1}{2\mathcal{C}_E} . \sum_{x\epsilon E}((\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2 + (\pi_A(x) - \pi_B(x))^2)}\right) \quad (8)$$

Obviously, when $r_A = r_B = 0$ i.e., A and B are standard PFS, then the proposed distances coincide with Pythagorean fuzzy distances (1)–(4).

Notation: Let $C_{PFS}(X)$ and $PFS(X)$ be the sets of all C-PFSs and set of all PFSs over the universe X, respectively. As mentioned above, $A \in PFS(X)$ iff $A \in$ C-PFS(X) and $r_a = 0$.

Theorem 1. For any $A_{r_A}, B_{r_B} \in C - PFS(X)$, that is A, B ∈ PFS(X) where $r_A, r_B \in [0,\sqrt{2}]$, the expressions (5)–(8) are well-defined metrics (distances).

Proof:

We need to show that the formulas

$H_2(A_{r_A}, B_{r_B})$, $H_3(A_{r_A}, B_{r_B})$, $E_2(A_{r_A}, B_{r_B})$, $E_3(A_{r_A}, B_{r_B})$,

stated in expressions (5)–(8) obey the three axioms for a metric from Definition1.

It is obvious that the expressions in (1)–(4) are well-defined distances over $PFS(X)$. Let D be any one of the distance measures H_2, H_3, E_2 or E_3. Hence, D is a metric in $PFS(X)$.

Since it is clear from the definition of C-PFSs, $A_{rA} = B_{rB}$ in C-PFS(X) iff $A = B$ in $PFS(X)$ and $r_A = r_B$. But $A = B$ in $PFS(X)$ iff $D(A, B) = 0$ and the sum of two nonnegative numbers is zero iff both numbers are equal to zero, therefore the validity of the first axiom for a metric is verified. The second axiom is obviously true since D is symmetric.

To prove the third property, consider a third C-PFS C_{rC} and verify

$$D(A_{r_A}, C_c) \le D(A_{r_A}, B_{r_B}) + D(B_{r_B}, C_{r_C}) \tag{9}$$

holds. We know that the triangle property for the PFSs A, B and C

$$D(A, C) \le D(A, B) + D(B, C) \tag{10}$$

holds. Also, the inequality $|x| + |y| \ge |x + y|$ for any three real numbers leads to

$$|r_C - r_A| \le |r_B - r_A| + |r_C - r_B| \tag{11}$$

for any values of r_A, r_B, r_C in $[0,\sqrt{2}]$. That is, the third axiom for distance is verified by summing up both sides of the last two inequalities (10) and (11).

Remark: *From the definition of the metrics* H_2, H_3 *and* E_2, E_3 *and since for any two PFS,* A, B *and* $x \in X : |\pi B(x) - \pi A(x)| \ge 0$, the following two inequalities are true.

- $H_2(A, B) \le H_3(A, B)$
- $E_2(A, B) \le E_3 A, B$

4.Numerical Illustrations

This section covers an Illustrative example to understand the concepts of the distances defined in this article for C-PFSs. A numerical example of a C-PFS with $X = \{x_1, x_2, x_3\}$, $C_E = 3$ and $A_{r_A}, B_{r_B} \in C\text{–}PFS(X)$ is pictured on Figure 3. For these C-PFSs A and B, $r_A = 0.2$ and $r_B = 0.5$ and the degrees of the corresponding elements x from the universe E are given in Table 1.

Table 1. Degrees of the element x C-PFSs A & B

$x \in E$	$\mu_A(x)$	$\mu_B(x)$	$\nu_A(x)$	$\nu_B(x)$	$\pi_A(x)$	$\pi_B(x)$
x_1	0.5	0.6	0.6245	0.8	0.3	0.5196
x_2	0.7	0.4	0.5916	0.6	0.8	0
x_3	0.9	0.2	0.3873	0.4	0.6	0.6928

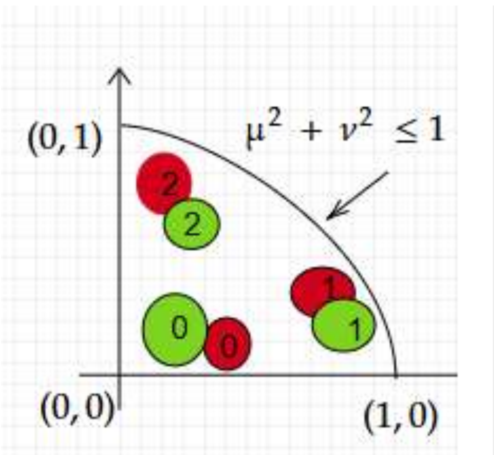

Figure 3. Pictorial representation of A_{rA}, B_{rB} ∈ C-PFS(X) with $E = \{0, 1, 2\}$, $C_E = 3$ and $r_A = 0.2$, $r_B = 0.5$.

Let us consider the two PFS, A_{rA} and B_{rB} from the previous example. Applying the corresponding formulas and the arbitrary specific values for A_{rA}, B_{rB} shows that

$$\frac{|r_A - r_B|}{\sqrt{2}} = \frac{|0.2 - 0.5|}{\sqrt{2}} = 0.2121$$

$H_2(A, B) = 0.3333$ and $E_2(A, B) = 0.1453$,
$H_3(A, B) = 0.4996$ and $E_3(A, B) = 0.2024$.

We calculate the values of the proposed metric formulae and find that,

$$H_2(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + H_2(A, B)\right) = 0.2627,$$

$$E_2(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + E_2(A, B)\right) = 0.1787,$$

$$H_3(A_{r_A}, B_{r_B})\frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + H_3(A, B)\right) = 0.3559,$$

$$E_3(A_{r_A}, B_{r_B}) = \frac{1}{2}\left(\frac{|r_A - r_B|}{\sqrt{2}} + E_3(A, B)\right) = 0.2073,$$

The comparison of results for the four different distances are shown in Table 2.

Table 2. Comparison of the distances for C-PFS.

D	$H_2(A_{rA}, B_{rB})$	$E_2(A_{rA}, B_{rB})$	$H_3(A_{rA}, B_{rB})$	$E_3(A_{rA}, B_{rB})$
$D(A_{rA}, B_{rB})$	0.2627	0.1787	0.3559	0.2073

The values of the different distances over A and B are compared. These values are plotted in Figure 3 that provides a visualization of C-PFS. In future, the authors are interested to introduce new distances for C-PFS. The distances over PFSs can be defined for C-PFSs as in Section 2.

5. Result and Discussion

Each one of the proposed distances satisfies the triangular inequality for any three arbitrary C-PFSs are shown below. Consider A_{rA}, B_{rB} ∈ C-PFS(X) as defined above and take another C-PFS C_{rC} ∈ C-PFS(X) with degrees of the element x from the universe X as given in Table 3.

Table 3. Degrees of the element x in C_{rC}.

$x \in E$	$\mu_C(x)$	$\nu_C(x)$	$\pi_C(x)$
x_1	0.5	0.7	0.5099
x_2	0.7	0.6	0.3873
x_3	0.8	0.5	0.3317

Applying the formulae from the previous section, the obtained values for the different distances between all combinations of pairs from $\{A_{rA}, B_{rB}, C_{rC}\}$∈ C-PFS(X) given in Table 4.

Table 4. Values of the distances H_2, E_2, H_3, E_3.

D	H_2	E_2	H_3	E_3
$D(A_{rA}, C_{rC})$	0.1644	0.1384	0.1896	0.1418
$D(A_{rA}, B_{rB})$	0.2727	0.1787	0.3559	0.2073
$D(B_{rB}, C_{rC})$	0.2311	0.1632	0.2943	0.1783

In the above table, if for any of the columns, if an arbitrary permutation of the row indices are considered. Let the corresponding values be denoted as a, b, c. Then, it can be easily checked that $a \leq b + c$, the triangular inequality of any of the proposed distances for the C-PFSs A_{rA}, B_{rB}, C_{rC} is validated. We can see that, considering column H_3 and the permutation of the row indices 3, 1, 2, then a = 0.2943, b = 0.3559, c = 0.1896 and 0.2943 < 0.3359 + 0.1896 = 0.5544 is validated.

6. Conclusion

The present paper discusses about the distance concepts in C-PFS. Here, some standard distance formulae for C-PFS were introduced. The distances that are formulated could be applied in more specific areas where the real time objects can be evaluated intensively when compared to an ordinary PFS. Also, the proposed distances are illustrated by means of suitable real time problems.

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